

# Models for Geometric CP Violation with Extra Dimensions

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## Abstract

In a recent paper, two of us (D.C. and R.N.M.) proposed a new way to break CP symmetry geometrically using orbifold projections. The mechanism can be realized in the five dimensional brane bulk picture. In this paper, we elaborate on this proposal and provide additional examples of models of this type. We also note the phenomenological implications of some of these models.

## 1 Introduction

There are many puzzles and mysteries in the highly successful standard model of electroweak interactions. One of the most prominent among them is the origin of CP violation observed in the kaon system and more recently perhaps, in the  $B$ -system. Another evidence for CP violation is in the domain of cosmology where there is evidence for asymmetry between matter and antimatter.

To introduce CP violation into gauge theories, one starts with the elementary field theoretic observation that complex couplings generally imply CP violation. In the standard model, these complex couplings[1] are introduced into the theory “by hand” and no insight is gained as to the origin of CP violation. A different way of introducing CP violation is to have the original Lagrangian to be CP conserving but to let the vacuum state break CP[2]. This phenomenon is known as spontaneous CP violation and it generically leads to a distinct picture for the early universe, where CP symmetry may be restored. An intriguing early suggestion in this context[3] is that the smallness of observed CP violation may be due to the fact that CP violation arises as a quantum effect. It is also worth remembering that a final resolution of the well known strong CP problem of QCD may depend on our true understanding of the origin of CP violation.

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In view of the significant role played by CP violation in a complete picture of particle physics, it is important to seek different ways to have a fundamental understanding of this phenomena. In a recent paper, two of us (D. C. and R. N. M.) [4] proposed a new geometric way to understand the origin of CP violation. The basic idea is to consider a theory in five space time dimensions (4+1) with the standard model residing in a 3+1 dimensional brane and have a degenerate pair of fields in the bulk (i.e. 4+1 dimensional space-time). It was then shown that if the two members of the degenerate pair are given asymmetric boundary conditions and their coupling to brane fields is suitably chosen, then it can result in CP violation. This is a fundamentally different way to introduce CP violation into particle physics from the ones known to date. While the models we present are in five dimensions, the mechanism can be easily generalized to arbitrary higher dimensions.

A key ingredient of this proposal is that there must be degeneracy of states which allows for a generalized definition of CP transformation. Often such degeneracy arises due to the very nature of five or higher dimensional space-time making the basic premise of these models very natural. For instance, in five dimensions, a fermion is necessarily four-component whereas in 4-dimensions, it can be 2-component; thus, the two 2-component spinors of the five dimensional 4-component spinor could be taken as the pair of particles. Another example is to consider supersymmetry in five dimensions; in the bulk, it automatically becomes an  $N = 2$  supersymmetry. If we consider a hypermultiplet of  $N = 2$ , it has two  $N = 1$  chiral multiplets. In the main body of the paper, we will give examples of both types and display how CP violation really arises.

Clearly, this way of breaking CP symmetry has another aesthetically appealing feature that now one can have an unified geometrical understanding of all forces such as gravitational, gauge as well as the origin of CP violation, a dream of many physicists ever since Einstein's general theory of relativity provided a successful description of gravitational forces. In this connection, it may be worth noting that in recent literature, there are several other examples of symmetry violation by geometrical effects e.g. parity [5], weak gauge symmetry [6] as well as the grand unification symmetries [7].

In Ref. [4], we discussed several examples of models where geometric CP violation arose from asymmetric boundary conditions in the bulk. In this paper we discuss several new examples and note some phenomenological implications of these models. We also discuss the general issue of the connection between complex phases and CP violations and emphasize that the presence of a complex phase in a theory does not necessarily mean CP violation, especially when there are degenerate particles. This basic observation is in some ways at the heart of our new mechanism.

Note that we do not purport to have a complete explanation of the origin of CP symmetry and its breaking. We assume that CP symmetry arises automatically out of some higher energy theory in higher dimensions such as string theory. We also cannot explain why nature may choose to compactify itself in an orbifold construction which violates the CP symmetry that was otherwise endorsed by the higher energy theory. These are hard questions whose understanding would require a better understanding of string theories themselves. However, it is very intriguing that given both possibilities, one can actually put the low energy CP violation that has been observed on our brane world into the context of a higher dimensional CP conserving world.

This paper is organized as follows: in Sec. 2 and 3, we present simple examples of models where a naive definition of CP transformation may suggest that the theory is CP

violating whereas a generalized definition clearly demonstrates that the theory is CP conserving. In Sec. 4, we give an example of a model that relates CP and P violation using similar ideas. In Sec. 5, we discuss another example where an apparently CP conserving theory turns out to be exactly CP conserving once one considers generalized CP transformations. In Sec. 6, we show how asymmetric boundary conditions in a compactified fifth dimensions can lead to CP violation and show that in certain models bulk Lorentz invariance forces the needed asymmetric boundary conditions. In Sec. 7, we discuss an extension of the standard model, where asymmetric boundary conditions arise automatically leading to the familiar Kobayashi-Maskawa model. Section 8 is devoted to a model where both P and CP have geometric origin and in Sec. 9, we elaborate on an example given in Ref.[4] where asymmetric boundary conditions implied by the  $N = 2$  supersymmetry in the bulk leads to a specific CP profile for MSSM. We discuss the phenomenological viability of this model. In Sec. 10, we give our concluding remarks.

## 2 Complex phases and CP violation

It is generally believed that a field theory that has complex phases leads to CP violating effects. The argument can be illustrated taking the example of a real scalar field  $\eta$  and a complex scalar field  $\phi$ . The relevant Lagrangian can be written as:

$$\mathcal{L}(\phi_1) = \lambda \eta \phi_1^* \phi_1 + m^2 \phi_1^* \phi_1 + (h \eta \bar{e}_L e_R + m_e \bar{e}_L e_R + H.c.) . \quad (1)$$

Note that even without  $\lambda$ , the complex phase in  $h$  cannot be removed once the mass  $m_e$  is made real. This is reflected in the electric dipole moment (EDM) of electron generated at one loop level, with  $\eta$  in the loop, which is proportional to  $\text{Im}(h^2 m_e)$ . There is an exception when  $h$  happens to be pure imaginary. In that case the Lagrangian is CP conserving without  $\lambda$ , and  $\eta$  can be defined to be CP odd. However with  $\lambda$  included, the theory is again CP violating because  $\lambda$  is real by hermiticity, and the  $\lambda$  interaction dictates the  $\eta$  has to be CP even, while the Yukawa term dictates the  $\eta$  to be CP odd. This CP violation is reflected in the two loop contribution[11] to the electric dipole moment[12] as in Fig.1. The effect is proportional to  $\lambda \text{Im}(h)$ .

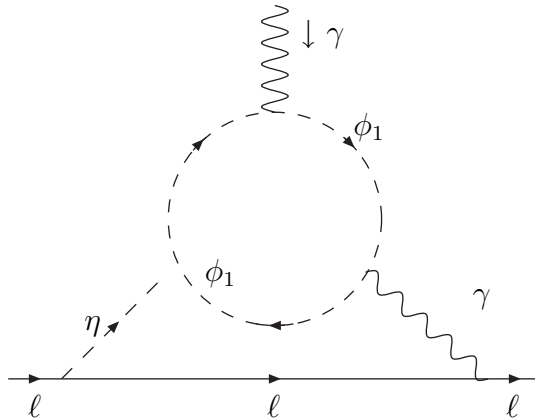


Fig. 1 The two-loop graph that contributes to the EDM of the lepton  $\ell$ .

Let us now extend this model by including another complex scalar field denoted by  $\phi_2$ , which is degenerate in mass with  $\phi_1$ . Consider the following Lagrangian:

$$\mathcal{L}(\phi_1, \phi_2) = \lambda\eta(\phi_1^*\phi_1 - \phi_2^*\phi_2) + m^2(\phi_1^*\phi_1 + \phi_2^*\phi_2) + (h\eta\bar{e}_L e_R + m_e\bar{e}_L e_R + H.c.) . \quad (2)$$

First, suppose  $h$  is purely imaginary. Then, if we defined CP transformations as usual i.e.  $\phi_i \rightarrow \phi_i^*$  under CP, it would appear that the model violates CP. However, if we define CP transformation in a more general manner i.e.  $\phi_1 \rightarrow \phi_2^*$ , then the Lagrangian is CP conserving. The Yukawa interaction dictates the  $\eta$  to be CP odd. With just one scalar  $\phi_1$  as in Eq.(1), this CP property is incompatible with the Higgs self-interaction. However with two degenerate  $\phi_i$ , in Eq.(2), it becomes possible to adjust the CP property of  $\phi_i$  such that  $\eta$  remains CP odd. It is easy to see that the two loop electric dipole diagram in Fig. 1, will now receive two contributions from  $\phi_{1,2}$  with opposite signs and equal contributions leading to zero EDM of the electron.

It is also easy to see from this example that if the two scalar fields had different masses, then there would be a CP violating contribution and one will have a CP violating theory. This is a different way to introduce CP violation into gauge theories. We will exploit this idea to propose new kinds of models of CP violation, including those with extra dimensions and discuss their implications.

Another manifestation of how CP violation is connected to the degeneracy of the  $\phi_i$  fields can be seen as follows. Suppose, we choose a potential for the CP-odd  $\eta$  field as

$$V(\eta) = m_\eta^2 \eta^2 + \lambda_\eta \eta^4 , \quad (3)$$

with  $m_\eta^2 > 0$ , then  $\langle \eta \rangle = 0$ . Thus vacuum also leaves CP as a good symmetry. Now if we take one loop effects for the case where there is mass degeneracy, the tadpole diagrams will cancel between  $\phi_{1,2}$  keeping the  $\langle \eta \rangle = 0$  VEV stable under radiative corrections. However once the mass degeneracy between  $\phi_{1,2}$  is removed, there will be a nonvanishing tadpole contribution leading to a VEV of the  $\eta$  field and one will produce the breakdown of CP invariance. Of course, one should note that while the VEV of  $\eta$  breaks CP symmetry, CP is strictly speaking not broken spontaneously. The loss of degeneracy of the scalar masses already breaks CP symmetry softly.

This provides us a new way to relate CP violation with other new phenomena in physics. For instance if the mass splitting between  $\phi_1$  and  $\phi_2$  arose from parity violation, as we show in a subsequent section, then parity violation become linked to CP violation providing a new way to understand the origin of CP violation. Similarly, one could relate this mass splitting to geometrical effects coming from extra dimensions, leading to a geometrical origin of CP violation.

Note also that in the above examples, the coupling constants all can be made real whenever a CP symmetry can be defined for the Lagrangians. However, one should note that having real coupling constants is a sufficient condition for CP symmetry but it is not necessary. In the appendix A, as well as in Sections 3 and 5, we provide some examples in which CP is conserved even in a theory in which there are some physical complex phases in the coupling constants.

### 3 Fermionic example

Next we shall consider models with fermions. First, consider a model with four left-handed chiral fermions  $f_1, f_2, f_3, f_4$  of charges  $+, -, +, -$  respectively and a real scalar  $\eta$ .

we define the C conjugated field of  $f$  as  $f^c = C\gamma_0 f^T$ . Thus

$$(f_2^T C f_1)^\dagger = f_1^T C f_2^c = f_2^T C f_1^c ,$$

$$(f_2^T C \sigma_{\mu\nu} f_1)^\dagger = f_1^T C \sigma_{\mu\nu} f_2^c = -f_2^T C \sigma_{\mu\nu} f_1^c .$$

Let us study the following Lagrangian

$$\begin{aligned} \mathcal{L}_f = & \lambda \eta (f_1^T C f_2 - f_3^{cT} C f_4^c) + \lambda^* \eta (f_1^{cT} C f_2^c - f_3^T C f_4) \\ & + \mu (f_1^T C f_2 + f_3^{cT} C f_4^c) + \mu^* (f_1^{cT} C f_2^c + f_3^T C f_4) \\ & + \Delta (f_1^T C f_4 + f_3^{cT} C f_2^c) + \Delta^* (f_1^{cT} C f_4^c + f_3^T C f_2) . \end{aligned} \quad (4)$$

We set up the system so that it respects the following CP symmetry,

$$f_1 \rightarrow f_3^c , \quad f_2 \rightarrow f_4^c , \quad \eta \rightarrow -\eta, \quad (5)$$

that is,  $\eta$  is CP-odd. One can make the mass parameters,  $\mu$  and  $\Delta$  real by changing the phase of the fermions. In this basis, it is clear that the complex phase of  $\lambda$  is a physical parameter. However, it has nothing to do with CP symmetry. It is interesting to note that there is a one loop contribution (with  $\eta$  boson in the loop) to the following dipole-moment operator,  $a f_2^T C \sigma_{\mu\nu} f_1$  as in Fig. 2, where the one loop coefficient  $a$  is complex and is proportional to  $\lambda^2 \mu^* e$ .

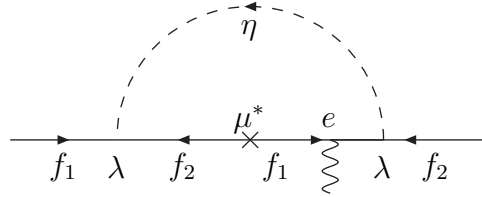


Fig. 2 The one-loop graph that contributes to electromagnetic dipole moment. The cross location denotes a mass insertion.

There is also a similar diagram that uses the  $\Delta$  mass insertion instead of  $\mu$ , it gives rise to the operator  $b f_4^T C \sigma_{\mu\nu} f_1$  with the one loop coefficient  $b$  which is real and is proportional to  $\lambda \lambda^* \Delta e$ .

Similarly, there are corresponding diagrams with  $f_1, f_2$  replaced by  $f_3, f_4$  which give rise to  $a^* f_4^T C \sigma_{\mu\nu} f_3$  and  $b f_2^T C \sigma_{\mu\nu} f_3$ .

So, one has one loop contribution to the magnetic dipole moments

$$\text{Re}(a)(f_2^T C \sigma_{\mu\nu} f_1 + f_1^{cT} C \sigma_{\mu\nu} f_2^c) + b(f_4^T C \sigma_{\mu\nu} f_1 + f_1^{cT} C \sigma_{\mu\nu} f_4^c) + (1, 2) \leftrightarrow (3, 4) , \quad (6)$$

as well as the electric dipole moments

$$i\text{Im}(a)[(f_2^T C \sigma_{\mu\nu} f_1 - f_1^{cT} C \sigma_{\mu\nu} f_2^c) - (f_4^T C \sigma_{\mu\nu} f_3 - f_3^{cT} C \sigma_{\mu\nu} f_4^c)] . \quad (7)$$

Note that in the limit that  $\Delta = 0$ , the Lagrangian has an  $U(1) \times U(1)$  flavor symmetry. In this limit, the  $(f_1, f_2)$  forms a Dirac pair as usual and so is  $(f_3, f_4)$  pair and the two pairs are degenerate in mass. In this sense, the EDM operators above is completely identical to those of ordinary fermion such as electron. The main difference is that, due to the degeneracy, these EDM's are not a direct signature of CP violation here because one can still define a conserved CP symmetry transforming the EDM of Dirac pair  $(f_1, f_2)$  into that of Dirac pair  $(f_3, f_4)$ .

The situation is a lot analogous to that of ammonia molecule which has double approximate degeneracy in its ground state with opposite parity property. The degeneracy is exact only when the tunneling between the two degenerate states is ignored. When tunneling is ignored, one can show that both states have non-zero EDM but of opposite sign without any breaking of CP symmetry. It is also interesting to note that when tunneling is included, the degeneracy is lifted, the original EDM becomes just the transitional moment between the two non-degenerate states. The role of the tunneling is played by  $\Delta$  in our field theory example.

When  $\Delta$  is nonzero, the mass eigenstates are

$$F_{\pm} = (f_1 \pm f_3)/\sqrt{2}, G_{\pm} = (f_2 \pm f_4)/\sqrt{2}. \quad (8)$$

They have nondegenerate mass terms

$$(\mu + \Delta)F_+^T C G_+ + (\mu - \Delta)F_-^T C G_- + H.c. . \quad (9)$$

Therefore  $(F_+, G_+)$  and  $(F_-, G_-)$  form two Dirac pairs  $H$  and  $K$  of masses  $\mu + \Delta$  and  $\mu - \Delta$  respectively,

$$\begin{aligned} H &= F_+ + G_+^c, \quad K = F_- + G_-^c, \\ -\bar{H}H &= -\bar{F}_+ G_+^c - \bar{G}_+^c F_+ = G_+^T C F_+ + H.c. , \\ -\bar{K}K &= -\bar{F}_- G_-^c - \bar{G}_-^c F_- = G_-^T C F_- + H.c. . \end{aligned}$$

The original EDM operators proportional to  $\text{Im}(a)$  in Eq. (7) can now be written as

$$i(\text{Im}a) [(G_-^T C \sigma_{\mu\nu} F_+ - F_+^{cT} C \sigma_{\mu\nu} G_-^c) + (G_+^T C \sigma_{\mu\nu} F_- - F_-^{cT} C \sigma_{\mu\nu} G_+^c)]. \quad (10)$$

With the property,  $\gamma_5 H = -F_+ + G_+^c$ ,  $\gamma_5 K = -F_- + G_-^c$ , we rewrite the above expression as the transitional electric dipole moment between two nondegenerate Dirac fields.

$$i(\text{Im}a)[\bar{K}\sigma_{\mu\nu}\gamma_5 H + \bar{H}\sigma_{\mu\nu}\gamma_5 K] .$$

It is also interesting to rewrite the original Yukawa coupling  $\lambda$  in this new basis:

$$\begin{aligned} \mathcal{L}_Y &= \text{Re}(\lambda) \quad \eta[(F_+^T C G_- + F_-^T C G_+) + H.c.] \\ &+ i \quad \text{Im}(\lambda) \quad \eta[(F_+^T C G_+ + F_-^T C G_-) - H.c.] , \\ &= -\text{Re}(\lambda) \quad \eta(\bar{K}H + \bar{H}K) + \text{Im}(\lambda)\eta(\bar{H}i\gamma_5 H + \bar{K}i\gamma_5 K) , \end{aligned} \quad (11)$$

reflecting the property that  $\eta$  is a CP odd scalar.

One may wonder what happens to the two loop Barr-Zee type contributions when a Yukawa coupling of  $\eta$  to the electron is introduced, such as  $ih\bar{e}\gamma_5 e$  as in Fig. 3. Each of  $f_1, f_2$  or  $f_3, f_4$  pairs contributes to the EDM of electron, however, the contributions come with opposite signs such that they cancel overall, as required by CP symmetry.

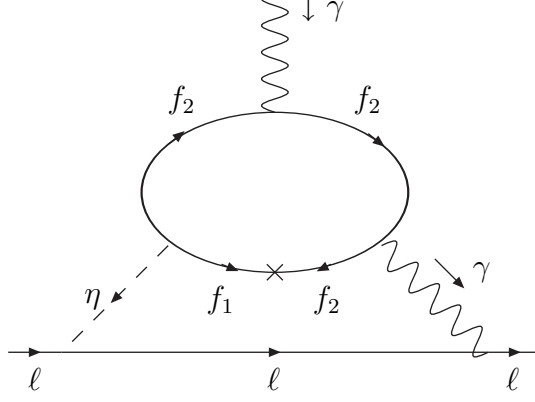


Fig. 3 The two-loop graph that contributes to EDM of lepton  $\ell$ . The cross location denotes a possible mass insertion.

## 4 Connecting P and CP violation

In this section, we present a left-right symmetric extension of the standard model where CP and P violation are connected to each other. The strategy is to start with a theory which prior to spontaneous symmetry breaking is both P and CP conserving. Using the analog of the bosonic model discussed in section 2, we show that once parity is broken, it also leads to CP violation.

We consider the usual left-right symmetric model[8] based on the gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  with fermion doublets  $Q \equiv (u, d)$  and  $\psi \equiv (\nu, e)$  assigned in a left-right symmetric manner. For the symmetry breaking, we choose doublets  $\chi_L(2, 1, 1)$  and  $\chi_R(1, 2, 1)$  and a bi-doublet  $\phi(2, 2, 0)$ . We add to this model a real pseudo-scalar, C-even field  $\eta$ . We will assume that the Lagrangian prior to spontaneous symmetry breaking is invariant under the following P and CP symmetries.

Under P, we have

$$\eta \leftrightarrow -\eta, \quad \phi \leftrightarrow \phi^\dagger, \quad \chi_L \leftrightarrow \chi_R, \quad Q_L \leftrightarrow Q_R. \quad (12)$$

Under charge conjugation C,

$$\eta \leftrightarrow \eta, \quad \phi \leftrightarrow \phi^T, \quad \chi_L \leftrightarrow \chi_L^*, \quad Q_L \leftrightarrow C\bar{Q}_R^T. \quad (13)$$

The Yukawa interactions are

$$f_{ij}\bar{Q}_{Li}\phi Q_{Rj} + g_{ij}\bar{Q}_{Li}\tilde{\phi} Q_{Rj} + H.c. \quad (14)$$

The parity symmetry P implies the the coupling matrices,  $f = f^\dagger$  and  $g = g^\dagger$ . The charge conjugation C implies  $f = f^T$  and  $g = g^T$ . Therefore both coupling matrices are real and symmetric.

Parity symmetry is broken when the parameters of the Higgs potential are chosen to be in a range such that  $\langle \chi_R^0 \rangle = v_R$  and  $\langle \chi_L \rangle = 0$ . We will show that this also leads to

CP violation. To see this let us write down the relevant part of the CP conserving Higgs potential:

$$V^{\text{Higgs}} = -\mu_+^2(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R) + \lambda_+(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)^2 + \lambda_-(\chi_L^\dagger\chi_L - \chi_R^\dagger\chi_R)^2 + \lambda\eta(\chi_L^\dagger\chi_L - \chi_R^\dagger\chi_R) + [m_\phi^2\text{Det } \phi + i\lambda'\eta\text{Det } \phi + H.c.] . \quad (15)$$

Note that in the limit of exact parity symmetry,  $\chi_{L,R}$  have same mass and the tadpole contributions to  $\langle\eta\rangle$  cancel leading to  $\langle\eta\rangle = 0$  and CP remains conserved. However as soon as parity is broken, the  $\chi_R$  disappears from the spectrum and  $\eta$  field has a nonzero VEV and CP violation occurs. It gets transmitted to the quark and lepton sector via the  $\eta\text{Det}\phi$  coupling, which now gives complex VEV for  $\phi$ . Thus CP and P violation get linked to each other. We do not elaborate on the phenomenology of this model but defer it to a future publication. We however note this way of relating P and CP violation is different from the one in [9]. We will also see how this scheme can be more naturally imbedded in higher dimensional theory in Section 8.

## 5 Duplicated standard model and CP violation

Another example, which also illustrates the point that complex phases need not necessarily imply CP violation is the model of Ref.[10]. This model uses the gauge group  $SU(2)_A \times SU(2)_B \times U(1)_Y$  and duplicates the field content of the standard model i.e.

$$q_L(2, 1)_{1/3} , u_R(1, 1)_{4/3} , d_R(1, 1)_{-2/3} , \psi_L(2, 1)_{-1} , e_R(1, 1)_{-2} ,$$

as the usual standard model fermions where  $(a, b)$  represents the  $SU(2)_A \times SU(2)_B$  representations; the duplicate fermions are

$$Q_L(1, 2)_{1/3} , U_R(1, 1)_{4/3} , D_R(1, 1)_{-2/3} , \Psi_L(1, 2)_{-1} , E_R(1, 1)_{-2} .$$

There are two Higgs doublets  $H_A$  and  $H_B$ , each a doublet under each group. It is assumed that under CP, the lower case fields transform into the upper case fields as:

$$\begin{aligned} q_L &\leftrightarrow \gamma_0 C \bar{Q}_L^T \\ u_R &\leftrightarrow \gamma_0 C \bar{U}_R^T \\ d_R &\leftrightarrow \gamma_0 C \bar{D}_R^T \\ H_A &\leftrightarrow H_B^* . \end{aligned} \quad (16)$$

and similarly for other fields. The CP invariant Yukawa coupling of the quarks can be written as:

$$\begin{aligned} \mathcal{L}'_Y &= \bar{q}_L H_A (h_d d_R + h'_d D_R) + \bar{q}_L \tilde{H}_A (h_u u_R + h'_u U_R) \\ &+ \bar{Q}_L H_B (h_d^* D_R + h_d'^* d_R) + \bar{Q}_L \tilde{H}_B (h_u^* U_R + h_u'^* u_R) + H.c. \end{aligned} \quad (17)$$

The coupling matrices  $h_{u,d}, h'_{u,d}$  are all complex and yet the theory is CP conserving. In fact as has been shown in Ref.[10], CP violation arises only if  $\langle H_A^0 \rangle \neq \langle H_B^0 \rangle$ . If the two VEV's become equal, both sets of gauge bosons have same mass and as a result, any linear combination of the two sets of gauge bosons is also an eigenstate and this enables one to get rid of all CP violating effects from the theory. In Section 7, we will illustrate how this scheme can be easily imbedded in a higher dimensional theory so that CP violation arises geometrically from the orbifold construction and results in an effective Standard KM model in the brane.



In this section, we show how the idea of the previous section can be used to connect CP violation to the geometry of space time. Consider for simplicity the complex scalar field model of Section 2 and assume that electron and  $\eta$  are brane fields whereas the fields  $\phi_{1,2}$  are bulk fields. Clearly the  $\eta$ - $\phi$  couplings in Eq. (2) involve the brane bulk coupling. Suppose we consider an  $S_1/Z_2$  orbifold where under  $Z_2$  symmetry  $y \rightarrow -y$ . We then expect that under the  $Z_2$  symmetry ( $R_P$ ),  $\phi_i \rightarrow \pm\phi_i$ . For even  $\phi$  fields the Fourier expansion will involve only the cosine modes whereas for the odd fields, only the sine modes will appear. If we assume that the brane is located at  $y = 0$ , then on this brane the odd  $\phi$  fields will vanish and the spectrum of the even and odd states will be asymmetric in their mass spectrum. In particular, this will make the  $\eta - \phi$  coupling obviously CP violating. Coming to the electric dipole moment of the electron in the toy model of section 2, the two loop diagrams do not suffer from complete cancellation and one gets a non-zero EDM for the electron.

In this example, one arbitrarily chooses the boundary conditions to get CP violation and an objection could be raised that CP violation is in some sense put in by hand although it is obviously connected to the geometry of the fifth dimension. It is however possible to construct models, where the asymmetric boundary conditions are dictated by kinematics of the fifth dimensions.

As an example, consider a model where there is a fermion in the bulk; its 5-dimensional kinetic energy can then be written in terms of the four dimensional fields as

$$i\bar{\psi}\gamma^\mu\partial_\mu\psi + (\bar{\psi}_L\partial_y\psi_R - \bar{\psi}_R\partial_y\psi_L) . \quad (18)$$

Due to the presence of the last term,  $Z_2$ -invariance implies that  $\psi_L$  and  $\psi_R$  have opposite  $Z_2$  parity. As a result, if one of them has even Fourier components i.e. cosines, the other field will necessarily have odd (sines) components and therefore vanish on the brane at  $y = 0$ . The asymmetry in spectrum necessary for CP violation will then arise more naturally. Similarly, if we have supersymmetry, then the bulk supersymmetry is  $N = 2$  type and in terms of the  $N = 1$  supersymmetry, an  $N = 2$  hypermultiplet has two  $N = 1$  chiral superfields ( $H, H^c$ ). In the effective  $N = 1$  Lagrangian, there is a term of the form  $\int dy H \partial_y H^c$  term. This endows the  $H$  and  $H^c$  fields with opposite  $Z_2$  parity. This in turn leads to asymmetric spectrum of fields and can be used to generate CP violation using our idea.

Below we give examples of models where an effective CP violating theory arises from the asymmetric boundary conditions described above.

## 7 KM model from asymmetric orbifold boundary conditions

To see how the familiar CKM model can be obtained from orbifold compactification, consider the model of the Sec. 5 in 5-dimensions, with the fifth dimension compactified on  $S_1/(Z_2 \times Z'_2)$ . In this case, there are four kinds of states denoted by

$$(+, +), (+, -), (-, +), (-, -) .$$

Except for the  $(+, +)$  states all other states have no zero modes and are therefore not visible at low energies ( $E \ll R^{-1}$ ). Note that the number of fermions doubles in the 5-dimensional model as compared to the 4-dimensional one i.e. the states now are  $(q_L, q_R)$  which are  $SU(2)_A$  doublets,  $(u_R, u_L); (d_L, d_R)$  are singlets; similarly for the leptons and the second set of quarks and leptons. We assign the following  $Z_2 \times Z'_2$  quantum numbers to the quarks and leptons (see table I):

Fields	$Z_2 \times Z'_2$ quantum number
$q_L, u_R, d_R, \psi_L, e_R, H_A$	$(+, +)$
$q_R, u_L, d_L, \psi_R, e_L, H_B$	$(-, -)$
$Q_L, U_L, D_L, \Psi_L, E_L$	$(+, -)$
$Q_R, U_R, D_R, \Psi_R, E_R$	$(-, +)$

The CP invariant Yukawa coupling of the quarks in five dimension can be written as:

$$\mathcal{L}'_Y = h_d \bar{q} H_A d + h_u \bar{q} \tilde{H}_A u + h_d^* \bar{Q} H_B D + h_u^* \bar{Q} \tilde{H}_B U + H.c. \quad (19)$$

Note that  $h'_d$  and  $h'_u$  in Eq.(16) are not allowed by  $Z_2 \times Z'_2$  symmetry. In the brane at  $y = 0$ , only the fields with  $(+, +)$  quantum numbers survive. This leads to the familiar CKM model. CP symmetry disappears because of the asymmetry in the spectrum created by the orbifold construction. The CP violating effect created by the complex phases in  $h_u$  and  $h_d$  was originally cancelled by the similar effects created by the CP conjugated states. However the asymmetry in the Kaluza Klein towers of the CP conjugated fermions destroy such cancellation.

## 8 Common geometric origin of P and CP violation

In this section, we use the left-right model of Sec. 4 to show both P and CP violation can have a common geometric origin. We start with the model of Sec. 4 in the brane and put a singlet neutrino  $\nu^B$  in the bulk. The brane field content is given by the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  gauge theory with fermion doublets  $Q \equiv (u, d)$  and  $\psi \equiv (\nu, e)$  assigned in a left-right symmetric manner and Higgs fields  $\chi_L(2, 1, 1)$  and  $\chi_R(1, 2, 1)$ , a bi-doublet  $\phi(2, 2, 0)$  and the P-odd and CP-odd field  $\eta$ . The fields transform under P and C as in Eq. (12).

The pure bulk part as well as the brane-bulk coupling terms are given by:

$$i\bar{\nu}^B \gamma^\mu \partial_\mu \nu^B + (\bar{\nu}_L^B \partial_y \nu_R^B - \bar{\nu}_R^B \partial_y \nu_L^B) + \int dy \delta(y) [\bar{\psi}_L \chi_L \nu^B + \bar{\psi}_R \chi_R \nu^B + H.c.] \quad (20)$$

As discussed in Sec. 6, the  $Z_2$  invariance of the bulk Lagrangian implies that under  $Z_2$  parity  $\nu_L^B$  and  $\nu_R^B$  have opposite parity. Let us therefore assume that  $\nu_L^B(x, -y) = \nu_L^B(x, y)$  whereas  $\nu_R^B(x, -y) = -\nu_R^B(x, y)$ . This implies that for a brane located at  $y = 0$ , the  $\nu_R^B$  field vanishes whereas the  $\nu_L^B$  field appaears full strength. The effective brane theory therefore is left-right asymmetric. As a result, the parity symmetry breaks. This asymmetrizes the masses of  $\chi_L$  and  $\chi_R$ . As already shown, under this circumstance, the  $\eta$  field will acquire a nonzero VEV and then lead to CP violation. The CP violating phase is transmitted to the fermions via the phases of the  $\langle \phi \rangle$ .

In order for the CP phase to manifest at low energies, one must have the two standard model Higgs doublets in the bi-doublet  $\phi$  survive at low energies below the  $W_R$  scale. If the  $W_R$  scale is in the TeV range, as possible in some extra dimensional models, then this does not require any fine tuning. On the other hand, if the  $W_R$  scale is high, some fine tuning may be needed for the purpose.

## 9 Profile of geometric CP violation in MSSM

In this section, we apply this new mechanism to generate CP violation in the minimal supersymmetric standard model (MSSM). For this purpose, we start with the usual MSSM field content in the brane (i.e.  $SU(2)_L \times U(1)_Y$  gauge group and superfields  $Q, L, u^c, d^c, e^c, H_u, H_d$ ) augmented by the inclusion of a single superfield, which will be the “messenger” of CP violation. In the bulk we will now have  $N = 2$  supersymmetry. We will have two  $N = 2$  hypermultiplets in the brane, denoted by its  $N = 1$  components  $(H_1, H_1^c; H_2, H_2^c)$ . Under CP symmetry, we assume the MSSM fields to transform as usual i.e.  $Q \rightarrow Q^c$ , etc. The rest of the fields transform as follows:

$$\begin{aligned}\eta &\rightarrow -\eta^* \\ H_1 &\rightarrow H_2^{c*} \\ H_2 &\rightarrow H_1^{c*} .\end{aligned}\tag{21}$$

We assume the theory prior to compactification to be CP symmetric so that the only phase in the theory is in the coupling of the  $\eta$  fields to the bulk fields  $H_{1,2}$ :

$$W_\eta = \eta(\lambda H_1 H_2 - \lambda^* H_1^c H_2^c) + M_1 \eta^2 + M_2 (H_1 H_2 + H_1^c H_2^c) ,\tag{22}$$

where  $M_{1,2}$  are masses expected to be of order of the fundamental scale of the theory. It is possible to have a theory where the mass parameters  $M_{1,2}$  and the familiar  $\mu$ -term could arise from a Kähler potential of the form[13]

$$S_K = \int d^4\theta \frac{S^\dagger}{M_{Pl}} [(H_1 H_2 + H_1^c H_2^c) + \beta H_u H_d] .\tag{23}$$

Now note that since the bulk kinetic energy leads to a term of the form[6]  $H \partial_y H^c$ , the required condition for CP violation i.e.  $H$  and  $H^c$  have opposite  $Z_2$  parity is automatically satisfied and CP violation in the brane will ensue rather naturally due to asymmetric spectrum of the bulk fields.

To see the profile of CP violation, let us write down the superpotential in the brane involving the  $\eta$  fields (the usual MSSM superpotential terms involving the MSSM fields are omitted for simplicity). To incorporate supersymmetry breaking, we have the usual hidden sector mechanisms in mind. We will use a singlet field  $S$  to implement the SUSY breaking by choosing  $\langle F_S \rangle = M^2 \approx (10^{11}) \text{ GeV}^2$ .

$$W_{brane} = (i\eta + M_{wk})(a + b \frac{S}{M_{Pl}}) H_u H_d .\tag{24}$$

We have not written terms that are suppressed by higher powers of  $M_{Pl}$  since their effect on CP violation is negligible.

CP violation in the MSSM arises when the field  $\eta$  acquires a nonzero VEV via the tadpole diagrams involving  $H_{1,2}$  fields. In the supersymmetric limit, due to the nonrenormalization theorem of supersymmetry,  $\langle \eta \rangle = 0$ . It is then easy to see that  $\langle \eta \rangle \simeq M_{susy}/(16\pi^2)$  if the parameter  $m_\eta$  is in the TeV scale. This leads to a profile of MSSM CP violation where the only CP violating terms are the  $\mu$  and the  $B\mu$  terms. All other CP violating phases in this model are extremely tiny due to Planck mass suppression. For instance, to get CP violation in the squark masses, one will have to write operators of type

$$\int d^4\theta \frac{\eta S S^\dagger}{M_{Pl}^3} Q^\dagger Q .$$

After the  $\eta$  field acquires a VEV, the resulting phase is of order  $10^{-16}$ , which is clearly too small.

By redefining one of the Higgs superfields, we can make the  $B\mu$  term real. So the only complex parameter in the theory is the  $\mu$  term. Furthermore, the CP phase is naturally of order  $10^{-2}$  due to the presence of the factor  $16\pi^2$  above. It could of course be larger if the mass parameters in the theory are adjusted (say somewhere between) 0.1 to 0.001, without going beyond usual naturalness requirements. There is no CP phase of the usual KM type in this model.

Let us briefly comment on whether such a model can explain observed CP violation in the kaon system. The only complex phase in the theory appears to be in the term

$$|\mu|e^{i\alpha}h_{d,ij}\tilde{Q}_iH_u^*\tilde{d}_j^c + |\mu|e^{i\alpha}h_{u,ij}\tilde{Q}_iH_d^*\tilde{u}_j^c ,$$

which gives rise to the squark  $LR$  mixing,

$$\delta_{LR,ij}^d = \frac{((A_d + |\mu|e^{i\alpha}\tan\beta)m^d)_{ij}}{M_{SUSY}^2} , \quad (25)$$

where  $m^d$  is the down quark mass matrix, and  $A_d$  is the trilinear soft supersymmetry breaking coupling matrix which is in general not the identity matrix at the weak scale, even if it may be the identical matrix at the supersymmetry breaking scale. It has been shown in Ref.[14] that for  $m_{\tilde{Q}} \sim m_{\tilde{G}} \simeq 500$  GeV, the constraints from  $\Delta m_K$ ,  $\varepsilon$  are respectively (for small phase  $\alpha$ )

$$\left(\text{Re}\delta_{LR,12}^d\right) \leq 4.4 \times 10^{-3} , \quad 2\left(\text{Re}\delta_{LR,12}^d\right)\left(\text{Im}\delta_{LR,12}^d\right) \leq 3.5 \times 10^{-4} .$$

If we saturate these values in our model, we then get  $|\varepsilon'/\varepsilon|$  to be  $1.4 \times 10^{-3}$  which is good agreement with experiments. The value of the electric dipole moment of neutron can be made smaller than the experimental limit  $11 \times 10^{-26}$  e·cm if  $|\text{Im}\delta_{LR,11}^d| \leq 3.0 \times 10^{-6}$  for  $m_{\tilde{Q}} \sim m_{\tilde{G}} \simeq 500$  GeV; and the effective  $\sin 2\beta$  parameter for  $B$ -decays to of order) 0.1 or less. These two predictions could be used to test this particular realization of our idea.

## 10 Conclusion

In this paper, we have elaborated on a novel mechanism for breaking CP symmetry, suggested recently by two of the authors where the compactified geometry of the fifth dimension played a crucial role. For this reason it was called geometric CP violation. The

essential idea is that the asymmetrization of the spectrum by orbifold conditions can lead to CP violating effects. Note that this is very different from many recent papers[15] on CP violation in models with extra dimensions in which CP violation is put into either a Higgs VEV on some other brane or a susy breaking VEV's. In our case, the mechanism is genuinely geometrical in nature. In this paper, we present several new realistic models that provide realizations of this idea and clarify the role of generalized CP transformations in implementing it. While we cannot explain how the CP symmetry arises in the fundamental higher dimensional theories, one application of this idea could be in the domain of string theories where, it is likely that the very large gauge symmetry which is the essence of the theory will be so constraining that there will be no room for a CP violation at the fundamental level. In that case, as pointed out here, one can imagine that the CP violation observed in the low energy theory is due to the particular compactification of the extra dimensions that somehow is favored by the dynamics of the fundamental theory.

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## Appendix A

In this appendix, we discuss the connection between complex phases in a Lagrangian and existence of CP violation. While it is generally true that any phase that cannot be removed by redefinition of complex fields in the theory is a physical phase and can lead to CP violation, we have found examples where a physical phase does not lead to CP violation. A simple example is provided in the following Lagrangian with two complex scalar fields  $\phi_{1,2}$  and one real scalar field  $\eta$ :

$$(\lambda\eta\phi_1^*\phi_2 + \delta m^2\phi_1^*\phi_2 + H.c.) + m_1^2\phi_1^*\phi_1 + m_2^2\phi_2^*\phi_2 , \quad (26)$$

where  $\lambda$  is a trilinear coupling. Note that by redefinition of the phases of the field  $\phi_{1,2}$ , one can make either  $\delta m^2$  or  $\lambda$  real but not both. The complex phase of  $\lambda$  in the basis in which  $\Delta m_2$  is real is clearly physical. However, it has nothing to do with CP violation. This can be seen by going to the mass eigenstate basis

$$\Phi_1 = +\phi_1 \cos \theta + \phi_2 \sin \theta , \quad (27)$$

$$\Phi_2 = -\phi_1 \sin \theta + \phi_2 \cos \theta . \quad (28)$$

In that case the most general Lagrangian can be written as

$$\lambda'\eta\Phi_1^*\Phi_2 + \lambda'^*\eta\Phi_2^*\Phi_1 + \lambda_1\eta\Phi_1^*\Phi_1 + \lambda_2\eta\Phi_2^*\Phi_2 + m_1^2\Phi_1^*\Phi_1 + m_2^2\Phi_2^*\Phi_2. \quad (29)$$

The phase of  $\lambda'^*$  is unphysical and can be removed by redefining the phase of, say,  $\Phi_2$ . Therefore in this basis, all the couplings are real and the theory has an obvious CP symmetry. Note that the corresponding couplings in Eq. (2) can be identify as  $\lambda_1 = -\lambda_2 = 2\text{Re}(\lambda)$  and  $\lambda' = i\text{Im}(\lambda)$ . The phase of  $\lambda$  is a physical parameter, but has nothing to do with CP violation. One can find a CP symmetry for Eq. (2) defined as  $\Phi_1 \rightarrow \Phi_1^*$ ,

$\Psi_2 \rightarrow -\Psi_2$  and  $\eta$  being CP even. Note however if one adds the coupling to the lepton in the form  $i h \bar{e} \gamma_5 e$  plus electron mass, then the theory become CP violating because the new coupling,  $h$ , forced the  $\eta$  to be CP odd instead. It will be reflected in a two loop contribution to electron EDM as in Fig.1 and the contribution is proportional to  $\lambda_1 \text{Im}(h)$  or  $\lambda_2 \text{Im}(h)$  depending on what is running in the inner loop. CP would be conserved if  $\lambda_1 = \lambda_2 = 0$ .

## Appendix B

In this appendix, we discuss a few elementary properties of fermions in five dimension. The ‘‘Lorentz’’ group in 5-dimensions is  $\text{SO}(4,1)$  and its algebra is specified by five  $\gamma$  matrices  $\gamma_{0,1,2,3,5}$ . We will choose the metric to be  $(+, -, -, -, -)$ . We choose a basis in which the  $\gamma$  matrices are given by (we use  $i, j = 1, 2, 3$  to be the known space indices; 0 stands for the time index and 5 for the 5th component; often in the text, we use  $y$  to denote the extra space index i.e.  $y = x_5$ ).

$$\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} ; \quad \gamma_0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix} ; \quad \gamma_5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} . \quad (30)$$

Each 4-dimensional Weyl spinor is a two-component object. However, the 5-dimensional spinor  $\Psi$  is necessarily a four-component one. Using  $\gamma_5$ , we can obtain from  $\Psi$  the chiral 4-dimensional spinors by the usual formulas i.e.  $\frac{1}{2}(1 \pm \gamma_5)\Psi \equiv \Psi_{L,R}$ .

One can define two kinds of parity transformations in this case: the usual P-parity under which  $\mathbf{x} \rightarrow -\mathbf{x}$  and a fifth dimensional parity (bulk parity or  $Z_2$  parity) under which  $y \rightarrow -y$ . As already mentioned in the text, invariance of the 5-dimensional kinetic energy term under bulk parity implies that under this  $\Psi_L$  and  $\Psi_R$  transform oppositely. As a result, the Dirac mass term  $\bar{\Psi}\Psi$  is not invariant under the bulk parity. However, the assignment of the absolute bulk parity is arbitrary. It therefore follows that if there are more than one bulk fermion, it is possible to assign bulk parities in such a way that one has mass terms involving the bulk fermions.

Turning to charge conjugation  $C$ , in the 4-dimensional case, it is usual to define it by the relation  $C\gamma_\mu C^{-1} = -\gamma_\mu^T$ . As a result, one can obtain  $C = \gamma_2\gamma_0$ . The  $C$  however commutes with  $\gamma_5$ . As a result, in five dimensions, one cannot use this definition of  $C$  and maintain 5-dimensional Lorentz invariance. The fifth component of the kinetic energy is not  $C$  invariant. One way to maintain this definition of charge conjugation in the five dimensional case is to simultaneously to transform the  $y \rightarrow -y$ [17]. This definition of  $C$  forbids the appearance of the the mass term for bulk fermions. In this case, one can forbid a Dirac mass term by  $Z_2$  invariance.

Another way is to define  $C = \gamma_2\gamma_0\gamma_5$ , which satisfies the property  $C\gamma_a C^{-1} = \gamma_a^T$  where  $a = 0, 1, 2, 3, 5$ . Note that 5-dimensional  $C$ -invariance also forbids the Dirac mass term involving  $\Psi$  but not the Majorana mass.

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